

Oberseminar

Numerik

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08.06.18

10:15 Uhr

Hilbertraum (05-432)

Staudingerweg 9, 55128 Mainz

„All Mach Number Second Order Semi-Implicit Scheme for the Euler Equations of Gas Dynamics“

Abstract: In this talk we present a simple strategy to construct semi-implicit schemes for Euler equations which are able to work on a wide range of Mach numbers. The schemes are based on staggered grid discretization in space in a fashion similar to the Nessyahu-Tadmor central scheme [4], which is second order accurate and avoids the needs of exact or approximate Riemann solvers. In the proposed schemes, acoustic waves are treated linearly implicitly, while material waves are treated explicitly. The resulting scheme is Asymptotic Preserving, i.e. it converges to a consistent scheme for the incompressible Euler equations as the Mach number vanishes. Because of the linear treatment of implicit terms, the schemes are quite efficient, especially for low Mach number flow. Second order schemes in time are constructed using Implicit-Explicit Runge-Kutta methods. Because of the staggered nature of the problem, second order discretization in time requires the use of Globally Stiffly Accurate schemes. A simplification is obtained by computing most of the stage values implicitly by a possibly non conservative predictor, while the conservative corrector guarantees that the overall scheme is conservative. A stability analysis is performed showing that the last stage of the scheme has to be implicit at the level of the numerical solution in order to avoid classical stability restrictions. Numerical convergence study is performed on various test problems, emphasizing the robustness and efficiency of the scheme. We obtain the same profiles found in the literature ([1, 3] for the isentropic case and [1, 2] for the general Euler system) for all Mach numbers. The procedure could be also extended to construct higher order schemes. Third order semi-implicit schemes on a non staggered grid are considered in [5]. Implicit treatment of boundary conditions will be subject of future investigation.

References

- [1] S. Noelle, G. Bispen, K. R. Arun, M. Lukacova-Medvidova, and C.-D. Munz. A weakly asymptotic preserving low Mach number scheme for the Euler equations of gas dynamics, *SIAM K. Sci. Comp.* 36(6), (2014), 989-1024.
- [2] P. Degond and M. Tang. All speed scheme for the low Mach number limit of the isentropic Euler equations, *Commun. Comput. Phys.*, 10(1):1f31, 2011.
- [3] J. Haack, S. Jin, and J.-G. Liu. An All-Speed Asymptotic-Preserving Method for the Isentropic Euler and Navier-Stokes Equations, *Commun. Comput. Phys.*, 12:955f980, 2012.
- [4] H. Nessyahu and E. Tadmor. Non-oscillatory Central Differencing for Hyperbolic Conservation Laws, *J. Comput. Phys.*, 87, 408-463 (1990).
- [5] S. Boscarino, J. Qiu, G. Russo, and T. Xiong. High order semi-implicit IMEX scheme for isentropic Euler system with all-Mach number. In preparation.

Hierzu sind alle herzlich eingeladen.

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