Numerical methods for fractional stochastic PDEs with applications to spatial statistics

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Abstract. *Many models in spatial statistics are based on Gaussian Matérn fields. Motivated by the relation between this class of Gaussian random fields and stochastic partial differential equations (PDEs), we consider the numerical solution of stochastic PDEs with additive spatial white noise on a bounded Euclidean domain* $\mathcal{D} \subset \mathbb{R}^d$*. The non-local differential operator is given by the fractional power* $\mathcal{A}^\beta$*,* $\beta > 0$*, of a second-order elliptic differential operator* $\mathcal{A}$*.*

We propose an approximation which combines recent Galerkin techniques for deterministic fractional-order PDEs with an efficient way to simulate white noise. Under minimal regularity assumptions on the differential operator $\mathcal{A}$*, we perform an error analysis for this approximation showing (i) strong convergence with respect to the norms on* $L_q(\Omega; L_2(\mathcal{D}))$*,* $L_q(\Omega; H^s(\mathcal{D}))$*,* $q \in (0, \infty)$*,* $s \in (0, 1]$*, (ii) weak convergence, and (iii) convergence of the covariance function at explicit and sharp rates. We furthermore address the computational complexity of the proposed method and prove that BPX preconditioning allows for generating one sample of the approximate solution at a certified accuracy and at costs in work and memory which are essentially linear in the degrees of freedom.*

For the motivating example of Gaussian Matérn fields, where $\mathcal{A} = -\Delta + \kappa^2$ *and* $\kappa \equiv \text{const.}$*, we perform several numerical experiments for various values of the fractional exponent* $\beta > 0$ *in dimensions* $d \in \{1, 2\}$*, which attest the theoretical results. Finally, we present a statistical application where we use the method for inference, i.e., for parameter estimation given data, and for spatial prediction. We observe that models which allow for a fractional exponent* $\beta > 0$ *exhibit a better predictive performance than non-fractional models.*