High-Order Methods: Interesting Aspects

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September 24, 2020

8.10.2020

Stability is an important property of numerical schemes for hyperbolic problems. In the linear case, L^2 stability is sought as a discrete analogue for the *a priori* energy estimates whereas in the context of nonlinear problems dominated by nonlinear convection terms, we seek *entropy stability* as a discrete analogue for the corresponding statement in the continuous setting.
In the first part of this talk, I will focus on entropy stability for several high-order schemes for hyperbolic conservation laws (e.g. discontinuous Galerkin (DG), Residual Distribution (RD)) and address recent progress in this field. Most of my investigations are done in the RD framework. Therefore, I start my talk by explaining the main idea of RD and its advantages. In [1], the author introduced a general tool to build entropy stable/conservative numerical schemes. In the following, we extend his idea and demonstrate some applications [2, 3] including a comparison to entropy stable flux differencing methods [4]. An outlook for future research finishes this part.
Besides entropy stability the preservation of other quantities is also important in numerical methods, e.g. the positivity of the density or pressure. In the second part of this talk, I focus on this issue and consider time-integration schemes for production-destruction systems (PDS). By the application of the modified Patankar trick, the authors of [5] were able to construct linear implicit, conservative and positivity preserving schemes of first and second order. Their methods are based on RK schemes and further extensions can be found in the literature [6]. However, until today they are limited up to a maximum of the third order. Using the modified Patankar trick instead in the Deferred Correction Method (DeC), we were able to construct conservative and positivity preserving modified Patankar (mPDeC) schemes of arbitrary high-order [7]. In the talk, I will introduce DeC as a time-integration scheme and specify advantages and relations. Finally, I will construct MPDeC schemes and verify their properties in numerical simulations.

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