Stochastic Galerkin Formulations for Hyperbolic Balance Laws

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The idea to represent stochastic processes by orthogonal polynomials has been employed in uncertainty quantification and inverse problems. This approach is known as stochastic Galerkin formulation with a generalized polynomial chaos (gPC) expansion. The gPC expansions of the stochastic input are substituted into the governing equations. Then, they are projected by a Galerkin method to obtain deterministic evolution equations for the gPC coefficients.
Applications of this procedure have been proven successful for diffusion and kinetic equations. So far, results for general hyperbolic systems are not available. A problem is posed by the fact that the deterministic Jacobian of the projected system differs from the random Jacobian of the original system and hence hyperbolicity is not guaranteed. Applications to hyperbolic conservation laws are in general limited to linear and scalar hyperbolic equations.
We analyze the loss of hyperbolicity for isothermal Euler and shallow water equations. In particular, hyperbolicity depends on the choice of gPC expansion and on the appropriate representation of positive physical quantities.