

The generalised solutions approach to isothermal multicomponent flows with Maxwell-Stefan diffusion

Pierre-Étienne Druet⁽¹⁾

(1) TU Darmstadt, Department of Mathematics, Research Group Analysis
Schlossgartenstraße 7, 64289 Darmstadt, Germany
e-mail: druet@mma.tu-darmstadt.de

For the mass diffusion fluxes $\mathbf{J}_1, \dots, \mathbf{J}_N$ in a multicomponent fluid with $N > 1$ substances, the modern constitutive Maxwell-Stefan equations read

$$(1) \quad - \sum_{k=1}^N f_{ik}(y_k \mathbf{J}_i - y_i \mathbf{J}_k) = \rho_i \left(\nabla \frac{\mu_i}{RT} - \sum_{k=1}^N y_k \nabla \frac{\mu_k}{RT} \right) \quad \text{for } i = 1, \dots, N.$$

where $(T, \rho_1, \dots, \rho_N)$ are the main thermodynamic state variables, i.e. the absolute temperature and the N partial mass densities, while $y_i = \rho_i / \sum_{k=1}^N \rho_k$ is the i^{th} mass fraction. For $i \neq k$, the phenomenological coefficient f_{ik} is typically positive, and it describes the intensity of the drag-force between two different species. The quotients $\mu_i/(RT)$ on the right-hand side are driving forces for the diffusion process, where μ_1, \dots, μ_N are usually called the chemical potentials and R is the gas constant. The so-called ideal mixtures are a common generalisation of the ideal gas law, characterised by the equations

$$(2) \quad \mu_i := g_i(T, p) + \frac{RT}{M_i} \ln x_i,$$

in which p is the thermodynamic pressure and x_i the i^{th} mole fraction, while M_1, \dots, M_N are the constant molar masses of the species. The general concave function g_i here denotes the Gibbs free enthalpy of the i^{th} constituent as pure substance.

If the fluid is not isobaric, the combination of (1) and (2) implies that pressure-gradients shall occur in the diffusion-flux. Moreover, a full mechanical description using momentum balance is necessary to compute the pressure. Hence, multicomponent diffusion in non isobaric fluids cannot be treated in the context of cross-diffusion systems.

In the talk, we consider an isothermal ideal multicomponent fluid, subject in $\Omega \times]0, \bar{\tau}[$ with $\Omega \subset \mathbb{R}^3$ bounded and $\bar{\tau} > 0$ to the equations

$$(3) \quad \partial_t \rho_i + \operatorname{div}(\rho_i \mathbf{v} + \mathbf{J}_i) = 0, \quad \text{for } i = 1, \dots, N,$$

$$(4) \quad \partial_t(\varrho \mathbf{v}) + \operatorname{div}(\varrho \mathbf{v} \otimes \mathbf{v} - \mathbb{S}) + \nabla p = \varrho \mathbf{b},$$

with the partial mass densities ρ_1, \dots, ρ_N of the species, and the velocity field $\mathbf{v} = (v_1, v_2, v_3)$ as main variables. The viscous stress tensor \mathbb{S} is assumed Newtonian with constant coefficients. The pressure and the mole densities are related to the main variables via

$$\sum_{i=1}^N \partial_p g_i(p) \rho_i = 1 \quad (\text{eq. of state}) \quad \text{and} \quad x_i = \frac{\rho_i}{M_i \sum_{j=1}^N (\rho_j / M_j)}.$$

Assuming general $\{f_{ik}\}$ and g_1, \dots, g_N , new mathematical challenges arise for the weak solution analysis of the coupled problem (1), (2), (3), (4). We shall focus on recent results on the resolvability in weak solution classes and, if time allow, also discuss the weak-strong uniqueness principle.